

Lesson K

Multiplying Integers Using Patterns

YOU WILL NEED

- a number line
- pencil crayons
- number line app (optional)



LEARNING GOAL

Model integer multiplication with number patterns.

LEARN ABOUT the Math

For 4 months, Liam's cellphone company mistakenly billed \$22 instead of \$25 for his smartphone. The company will bill Liam the extra money he owes on his next bill.



What integer shows how Liam's bank balance will change?



Example 1

Using a number pattern

I used a multiplication pattern to calculate the change.

Liam's Solution

$$(+4) \times (-3) = \text{change}$$

$$(+4) \times (+3) = (+12)$$

$$(+4) \times (+2) = (+8)$$

$$(+4) \times (+1) = (+4)$$

$$(+4) \times 0 = 0$$

$$(+4) \times (-1) = (-4)$$

$$(+4) \times (-2) = (-8)$$

$$(+4) \times (-3) = (-12)$$

My bank balance will change by $-\$12$.

I used $+4$ to represent 4 months.

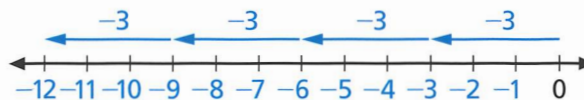
I multiplied by -3 to show the extra $\$3$ that I owe for each month.

I know $(+4) \times (+3) = (+12)$. I used this to start a multiplication pattern.

I stopped the pattern at $(+4) \times (-3) = (-12)$.

Reflecting

A. How does this number line show Liam's answer is correct?



B. Why do $(+4) \times (+3)$ and $(+4) \times (-3)$ have products with opposite signs? Use the number line to explain.

WORK WITH the Math

Example 2 | Modelling a negative number of groups

Calculate $(-4) \times (+2)$ using the commutative property.

Solution

$$\begin{aligned}(-4) \times (+2) &= (+2) \times (-4) \\ &= (-4) + (-4) \\ &= (-8)\end{aligned}$$

Use the commutative property to rewrite the multiplication with a positive number of groups. Then write the repeated addition.

$$(+2) \times (-4) = (-8), \text{ so } (-4) \times (+2) = (-8)$$

Communication | Tip

You can show multiplication with brackets instead of a \times symbol. For example, $(-4) \times (+5) = (-4)(+5)$.

Example 3 | Multiplying 2 negative integers

Use a number pattern to calculate $(-4)(-5)$.

Solution

$$(-4)(-5) = (20)$$

Use $4 \times 5 = 20$ to determine the number part of the product.

$$\begin{aligned} (+2)(-5) &= (-10) \\ (+1)(-5) &= (-5) \\ (0)(-5) &= (0) \\ (-1)(-5) &= (+5) \\ (-2)(-5) &= (+10) \\ (-3)(-5) &= (+15) \\ (-4)(-5) &= (+20) \end{aligned}$$

Use a pattern to determine the sign of the product.

Build the pattern up and down from $(0)(-5)$.
Extend the pattern to $(-4)(-5)$.

A Checking

1. a) Write the multiplication pattern.

- The pattern starts at $(+2) \times (+5) = (+10)$.
- The first integer decreases by 1 each time.
- Stop when the product is -10 .

- b) What pattern do you see in the products?
c) Use the product pattern to calculate $(-5) \times (+5)$.

2. Calculate. Explain how you determined the sign.

- a) $(-6) \times (+10)$ c) $(-10) \times (-10)$
b) $(+8) \times (-10)$ d) $(-3) \times (+10)$

3. What sign will each product have?

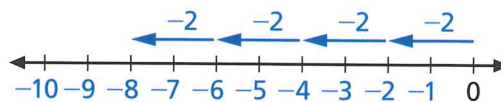
$$\begin{array}{cc} (+_)\times(+_)\quad (-_)\times(+_)\quad \\ (+_)\times(-_)\quad (-_)\times(-_)\quad \end{array}$$

B Practising

4. Calculate. Choose 1 multiplication, and explain how you calculated.

- a) $(+6) \times (-12)$ c) $(-18) \times (+3)$
b) $(+21) \times (-4)$ d) $(-3) \times (-15)$

5. a) Write the multiplication equation for the number line.



- b) Use the commutative property to write a different multiplication equation with the same product.

6. Fiona withdraws \$25 from her bank account 8 times.

- a) Write an integer multiplication equation for this situation.
b) Explain how you decided what sign to use for the product.



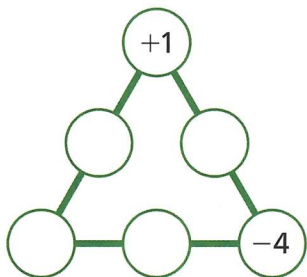
7. Bernice and Max chose signs for these 2 integers and multiplied the integers. Bernice's product was greater than Max's product. What signs could Bernice have used? Explain.

$$(-8) \times (-5)$$

8. 2 integers have a product of +8. What could the integers be? Write all the possibilities.
9. a) When you multiply an integer by +1, what is the result?
 b) When you multiply an integer by -1, what is the result?
 c) Predict the sign of each product. How did you predict?

$$(-30) \times (+50) \quad (+30) \times (-50) \quad (-30) \times (-50)$$

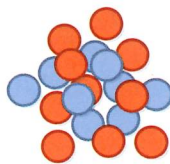
10. The position of an elevator changes by -4 m for every floor it goes down. What integer describes each drop?
- a) down 0 floors c) down 11 floors
 b) down 6 floors d) down 15 floors



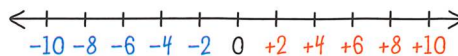
11. a) Solve this puzzle.

The product of the 3 integers on each side of this triangle is -24 . Each circle has a different integer.

- b) Compare solution strategies with a partner. What did you have to consider to solve the puzzle?
12. Choose a model to explain to a partner why the product of 2 negative integers is positive. Then listen to your partner's explanation. How were your explanations different?



$$\begin{aligned} (+2) \times (-3) &= (-6) \\ (+1) \times (-3) &= (-3) \\ 0 \times (-3) &= 0 \end{aligned}$$



Multiplying Integers (A)

Find each product.

$(-6) \times 0 =$	$7 \times 3 =$	$6 \times (-10) =$	$(-3) \times (-5) =$
$8 \times (-2) =$	$(-4) \times (-10) =$	$10 \times (-3) =$	$3 \times 5 =$
$9 \times (-4) =$	$10 \times 4 =$	$10 \times (-4) =$	$5 \times 9 =$
$0 \times (-10) =$	$11 \times 11 =$	$2 \times 3 =$	$(-4) \times (-12) =$
$(-4) \times (-6) =$	$(-10) \times (-2) =$	$3 \times 12 =$	$4 \times 7 =$
$2 \times 4 =$	$3 \times (-3) =$	$(-12) \times (-12) =$	$(-9) \times 5 =$
$9 \times (-7) =$	$9 \times 8 =$	$(-1) \times 10 =$	$(-1) \times (-2) =$
$4 \times (-12) =$	$(-6) \times (-5) =$	$10 \times (-1) =$	$(-7) \times (-9) =$
$7 \times 4 =$	$6 \times (-5) =$	$9 \times (-12) =$	$8 \times 1 =$
$(-2) \times 1 =$	$(-11) \times 2 =$	$12 \times 3 =$	$(-4) \times 3 =$
$7 \times (-8) =$	$11 \times 2 =$	$7 \times 11 =$	$(-9) \times (-12) =$
$(-12) \times 7 =$	$4 \times 10 =$	$8 \times 5 =$	$0 \times 3 =$
$11 \times 7 =$	$1 \times (-6) =$	$(-11) \times 4 =$	$0 \times (-6) =$
$11 \times (-9) =$	$4 \times (-2) =$	$2 \times (-11) =$	$(-5) \times 12 =$
$(-3) \times 1 =$	$(-1) \times 11 =$	$7 \times (-10) =$	$(-7) \times (-3) =$
$(-11) \times (-11) =$	$8 \times 4 =$	$(-3) \times 12 =$	$(-10) \times (-6) =$
$2 \times 7 =$	$(-5) \times 10 =$	$(-7) \times 5 =$	$(-2) \times 2 =$
$6 \times (-4) =$	$10 \times (-11) =$	$(-4) \times (-3) =$	$(-8) \times (-2) =$
$2 \times 12 =$	$(-4) \times 1 =$	$(-4) \times 7 =$	$(-1) \times 5 =$
$4 \times (-8) =$	$(-2) \times (-11) =$	$(-10) \times 7 =$	$(-8) \times 9 =$
$(-1) \times 2 =$	$(-9) \times (-8) =$	$1 \times 5 =$	$(-6) \times 12 =$
$(-10) \times (-4) =$	$(-11) \times (-10) =$	$1 \times (-12) =$	$3 \times (-7) =$
$(-3) \times (-4) =$	$8 \times 12 =$	$2 \times (-8) =$	$0 \times 8 =$
$5 \times (-7) =$	$0 \times 11 =$	$(-10) \times 10 =$	$(-8) \times 0 =$
$4 \times (-7) =$	$11 \times 1 =$	$(-3) \times 8 =$	$(-2) \times (-10) =$